EXPORTING ENGINEERING TECHNOLOGY PRACTICE TO ENHANCE PRE-COLLEGE MATHEMATICS LEARNING

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Abstract

This presentation combines two theoretical perspectives. The first perspective, from engineering education, emphasizes the importance of communicating essential knowledge to non-engineers. The second theoretical perspective comes from the mathematics education research literature. It is well established that students may be able to recall certain facts, but fail to use those facts in solving novel problems. In many cases, students do not even recognize that solving such problems is important.

This presentation describes how undergraduate engineering majors designed robots for a class project. These robots are low cost, built with mass produced low precision parts. Calibration and error-correction techniques in software and hardware are used to enhance their precision. These robots are designed to draw geometric art, and in the process, teach Mathematics to high school students. A detailed analysis of the necessary subject matter knowledge is provided here. Furthermore, a high school mathematics teacher has examined the robots from the perspective of the classroom. Methods to motivate learners and enhance instruction are described.

Students are interested in real world problems. When students see the robots graph linear functions, they have an opportunity to analyze the relationship between algebra and geometry. Both linear functions and the Pythagorean theorem are central to high school mathematics. There is an enormous difference between drawing a line on graph paper versus writing a program for the robot to travel along a line in the real world with real error. This has the added benefit of examining the important topic of estimation.

Bringing engineering technology into the classroom includes benefits at both levels. College students learn to communicate the results of their course work to an interested audience, with less specialized knowledge. High school students benefit by learning to apply knowledge of mathematics in novel ways to real world problems. As a result, they can develop a richer knowledge base for their mathematics education.

Introduction

One important goal of engineering education is to enhance the communication of essential knowledge to non-engineers. “Even though we are able to create complex objects that could save lives or millions of dollars, it is all worth nothing if we cannot communicate our ideas or work with others.” **[1]**

In the mathematics education research literature, it is well-established that students may be able to recall certain facts, but fail to use those facts in solving novel problems. In many cases, students do not even recognize that solving such problems is important. Students often “give clear evidence of knowing certain mathematics but then proceed to act as if they are completely ignorant of it." **[2]**

Bringing engineering technology into the mathematics classroom can help students understand the subject matter more deeply than in traditional mathematics instruction. Students can learn that "Mathematics has two faces; it is the rigorous science of Euclid, but it is also an experimental science: mathematics in the process of being invented." **[3]** The effectiveness of the teachers’ work can be determined by balancing these two aspects of mathematics. We need to help expand our students’ mathematical toolbox: they need to learn the rules, concepts and formulas to solve mathematical problems. Students, however, who only encounter the abstract nature of mathematics are often describing it as a boring and dry subject and state that they never learn anything that they could use in real life. When practicum and pure thought are inextricably intertwined, that is real math instruction. Teaching math with robots combines these two sides of mathematics.

Real life word problems are extremely difficult for students because they have to read, understand, and visualize the problems. Students have to sort through words and information and decide which one is relevant and necessary to solve the problem. Students in general are used to one step problem solving and are perplexed when they encounter a problem that they have to plan, define the variables, and step by step solve. Application of mathematical skills is much more complicated than practicing the skills in isolation.

Problem solving is not only part of math instruction but an extremely vital part of our everyday life. Without a flexible base from which to work, students may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, and explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut.

Following the spirit of the Core Curriculum, we emphasize “solving real world and mathematical problems.” **[4]** In the classroom, many students do not see the point of mathematical problems. The advantage of robotics is that these problems can be acted out visually, and controlled in a hands-on manner. Furthermore, we will examine the effect of real world error. We are making the problem more accessible to students, but not by over-simplification. In fact, we are asking students to explore the problem in greater depth.

Students are more likely to appreciate problem solving when real-world examples are included. When students see the robots graph linear functions, they have an opportunity to analyze the relationship between algebra and geometry. Both linear functions and the Pythagorean theorem are central to high school mathematics.

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Conceptual errors and real world errors

Conceptual errors involve a misunderstanding of a mathematical idea, such as incorrectly believing that the hypotenuse of a right triangle can be longer than the sum of the two legs. When students examine conceptual errors, the process enhances mathematical reasoning and lead to deeper learning. **[5] [6]** Student errors are an inherent part of the learning process, and analyzing errors can provide learning opportunities. Errors cannot be removed simply by teaching the correct information; the student needs to understand what has gone wrong. Instructional approaches should capitalize on errors by focusing directly on analyzing and discussing conceptual errors. Students should be exposed to correct answers and to conceptual errors.

Errors involved in line-drawing with robots

Real world errors are inherent and cannot be avoided. Low cost robots will need to use mass manufactured parts in their kits. That brings in the issue of manufacturing variability. For example, a wheel may not be precisely 2.5 cm in radius; it might have a tolerance of + 5% in the radius. If one were to determine the distance traveled as being equivalent to a number of full and fractional turns, this tolerance will carry over to the distance traversed as well. A fractional or full turn can be estimated with the help of optical encoders. Such encoders will generate a pulse for a certain degrees of rotation of the wheel. However, there is finite resolution associated with this. As a consequence, there might be errors involved in quantifying fractional turns, which will add further ambiguity to the distance traversed. In our elucidation of the Pythagorean theorem, the robot needs to make 45 and 90 degree turns. Such turns can also benefit from optical encoders, since it is a differential drive to the two sides of wheels that will bring about the physical turn. There could be errors in creating physical turns by these angles, furthering the ambiguity.

These three types of errors may be considered to be systematic errors, but random with respect to each other. Thus, the total error from these errors is not an arithmetic sum, but it will be based on the RMS (root mean square) sum, and will be less. We calculated that the error in the estimation of the distance traversed by the robots built with our kits will be approximately 10% or slightly higher. This, however, is strictly based on the error compensation one can achieve with hardware alone. However, this can be improved in two distinct ways: (1) positional adjustment once the destination is reached, with the aid of an ultrasound range sensor; and (2) use of predictive algorithms in software to estimate fractional turns to a better accuracy than is feasible with feedback pulses from the optical encoder itself.

An Unmet Need: Bringing Robotics into the Classroom

This project started when we learned about a local school that had a robotics club but no robots. Students, both and girls, met once a week to discuss how to finance their club, but made no headway. Commercially available robots were beyond their means. We wanted a simple low cost solution that they could easily follow and build upon. We also wanted them to sustain their program by their own innovation and creativity.

At some schools, teachers use Lego [7] and VEX [8], and these are very popular. However, we believe that they are expensive, self-contained, and too sophisticated to be used for Mathematics education in all K-12 schools. Our intent here is to use and create open source tools, to keep the cost low, but also to make the process more transparent –that is, students, teachers, and engineers can investigate, learn from, and improve upon, both hardware and software implementations.

In fall 2011, a course was offered on embedded robotics. The class had 21 students, equally divided among computer science, computer engineering, and electrical engineering students. Their goal was to develop algorithms and hardware for drawing geometric shapes (and hence create robotic art) given a set of low cost and open source components. **[9] [10]** The students used a pen and large paper canvas (6' x 6') to draw these figures.

Robotics capture the imagination of many young people. However, the high cost, super-sophistication, proprietary nature, and limited programmability of commercially available robots, tend to dampen such interest. We believe that simple, low cost robots that are easy to control and manipulate will reverse this trend. Our goal is to develop low cost robotic kits that are incrementally acquirable. Open sourcing will allow the cost to fall further. Thus, a school can initially acquire a few robots (or build them at still lower costs) at a cost of less than $150 each, and incrementally add a few more robots every six to twelve months. Even a few robots will be able to give an adequate educational environment for the students. Availability of low-cost downloadable applications may persuade the pooling of robotic resources among schools to host competitions and design their own new puzzles, which can then be marketed to generate revenue. Our experience with Android smart phone applications leads us to believe that this is feasible. **[11] [12]** In this era of reduced budgeting, creative solutions are warranted; and further, in this era of heightened global competition, we need to emphasize innovation and entrepreneurship. We thus expect a trend towards low cost and open source solutions that will benefit all, not just a few major business entities. This is a welcome change and provides a way out in the changing world with pressing economic challenges. We also perceive the evolution of a healthy and social environment since such a robotic exercise is not solitary (or a virtual on-line game), as many video games are, and can be held in open space as a community activity. Further, it will not be just limited to robotic enthusiasts.

As mentioned earlier, the LEGO Group provides educational solutions that allow children to learn as they play. LEGO MINDSTORMS is used by thousands of teachers to teach 21st century skills to millions of students. The Carnegie Mellon / LEGO Education partnership is dedicated to using robotics as the motivator to 'simply teach the complex'. **[7]** The VEX Robotics system allows students and hobbyists to build "real" robots. VEX kits provide the whole solution or can be integrated with large and small motors, pneumatic actuators, sensors, electronic components, composites, and a student's imagination to engineer a unique mechatronic solution. **[8]**

The long-term goals of this project are to use the open source tools of Arduino and extend it to robotics. There is a need for an open source approach to help all to learn from each other and increase the number of lessons and examples. If the students at a school develop innovative ideas, they can use the open source market to generate some revenue and help acquire more robots. Lego's sophistication can raise students' expectations to unrealistic level -- and prevent them from choosing cost-effective solutions. In contrast, the robots used in this study encourage students to consider more economical solutions, which also consume less power, are more transparent, and are better targeted for educational purposes.

Embedded Robotics Course

Our experiences from fall 2011 has allowed us to draw up a lesson plan that involves the introduction of one component at a time: Draw a line (use of stepper motors); vary PWM (pulse width modulation) to slow and speed up the motion; draw a straight line on a narrower grid (with the aid of parallel IR (Infra-red) reflective walls and side-mounted IR range sensors on the robots; make the robot stop after a given distance with the front-mounted US (ultrasound) range sensor and a reflective wall in the front; and draw a rectangle, one line at a time.

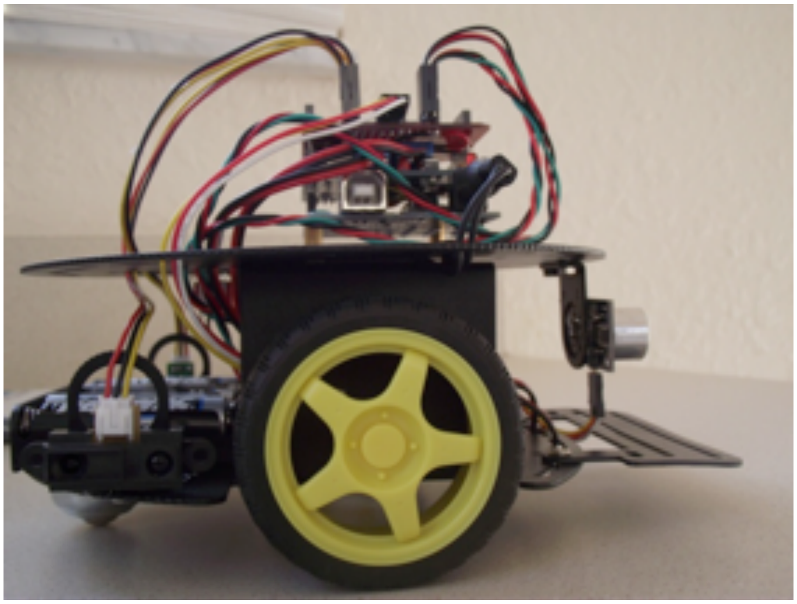

Figure 1: A robot constructed in the class this semester.

In a course in Fall 2011 engineering students explored ways to create geometric shapes using low cost robotic systems. Figure 1 shows the robot being used this semester with 9th grade students (more details below).

There were many engineering challenges for students, such as limited range sensing, slippage on paper, drag due to a heavy pen, four-wheel drive, battery drainage, etc. Students were given suggestions for use of debouncing, interrupts, and feedback. **[13]** We have learned from these experiences and now have improved the robot (see Figure 1) and accessories to overcome them.

Results of Pilot Studies

This section describes the results of three projects from the Embedded Robotics class. These projects are:

1. Graphing a Linear Function on a Graphics Screen

2. Drawing a Triangle with a Robot

3. Creating Computer Art on both a Graphics Screen and with a Robot

Graphing a Linear Function On a Graphics Screen

One example was a computer program that drew a graph of the function y = x (Figure 2).

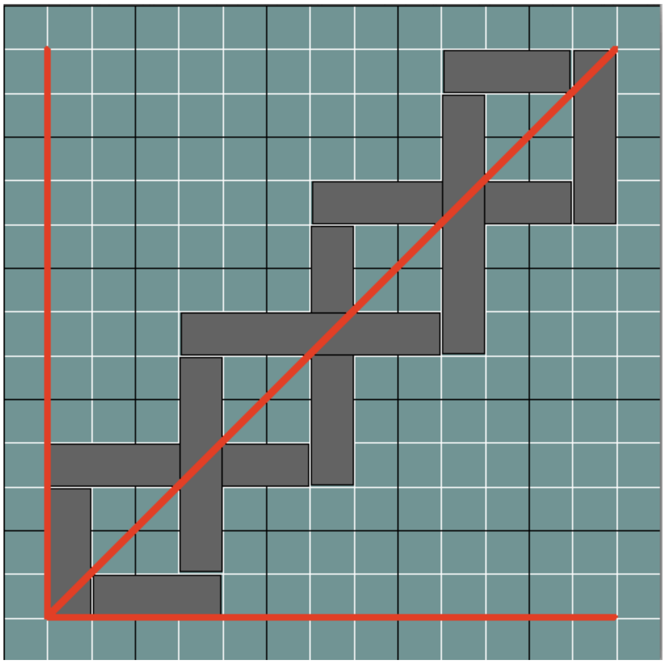

Figure 2: A computer graphics drawing, showing the function, y = x.

Drawing a Triangle With a Robot

Figure 3 shows a right triangle as drawn by a robot.
Getting a robot to draw a triangle requires much more knowledge than that typically acquired by high school students in a geometry course.

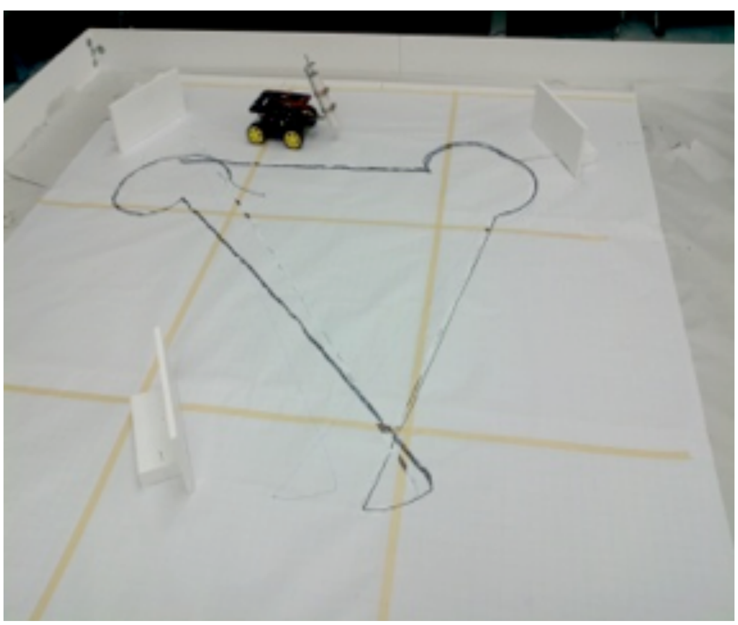

Figure 3: A triangle drawn by a robot.

Robots in the mathematics classroom

The following example shows how enhanced visualization with hands-on models play out real world problems. Following the spirit of the Core Curriculum, we emphasize solving real world and mathematical problems. This example is directly related to the specific subject matter standards: the Pythagorean theorem. In the classroom, many students do not see the point of such problems. The advantage of robotics is that these problems can be acted out visually, and controlled in a hands-on manner.

The following example shows how enhanced visualization with hands-on models play out real world problems. Following the spirit of the Core Curriculum, we emphasize "solving real world and mathematical problems."

Problem: Tim leaves school and walks east for 3 blocks. He then turns left and walks north for 4 blocks. Tamara leaves school and walks 6 blocks west. Then she turns left and walks 8 blocks south. How far does Tamara live from Tim?

Solution: Tim is walking the legs of a right triangle. Since the legs are 3 and 4, we know that the hypotenuse must be 5. Therefore Tim's total distance walked is (3 + 4 = 7). However, his distance from school is direct (therefore, 5). Likewise, Tamara is walking a (geometrically) similar right triangle, with legs are 6 and 8, we know that the hypotenuse must be 10. Therefore Tamara's total distance walked is (6 + 8 = 14). However, his distance from school is direct (therefore, 10). Since Tim and Tamara walked in completely the opposite direction, the distance between the houses is 9 blocks west and 12 blocks south. These are also legs of a right triangle. Since the legs are 9 and 12, we know that the hypotenuse must be 15. Note that this is also the sum of the distances from school: 5 + 10 = 15.

In a mathematical problem involving a right triangle with legs 6 and 8 units long, the hypotenuse must be exactly 10. With the robots, the problem will be solved in the real world. For example, in a mathematical problem with a right triangle whose legs are 6 and 8, the hypotenuse must be exactly 10. But in real world problems, things are never so neat.

There are benefits to using robots, especially increased interest in the process. Robots work in the real world, and do not yield mathematically exact results. The mathematical problem provides a guide to the real world problem. If the solution to the mathematical problem is 10, and the real world measurement is 9.3, then students will learn that errors always occur in working with actual materials and will accept the result as "close enough."

However, if the answer is 17.1, then something is conceptually incorrect. This requires much more involvement in the problem than students typically have. Students must understand both the mathematical problem and the real world problem. The robots require the student to monitor the success of the problem solving process, as the robot illustrates the solution to the problem. Also, students can see the accuracy of their solution by comparing the obtained solution with the actual location of the robots, which is the basis of the error checking process. Both monitoring and error checking enhance mathematical problem solving.

Creating Computer Art

One group (Adam Corbin, Cody Neuburger,and Nannette Suarez) became interested in a much more sophisticated geometric design, a nested multi-dodecagon. Figure 4 shows the output of their program that drew the design on a computer graphics screen.

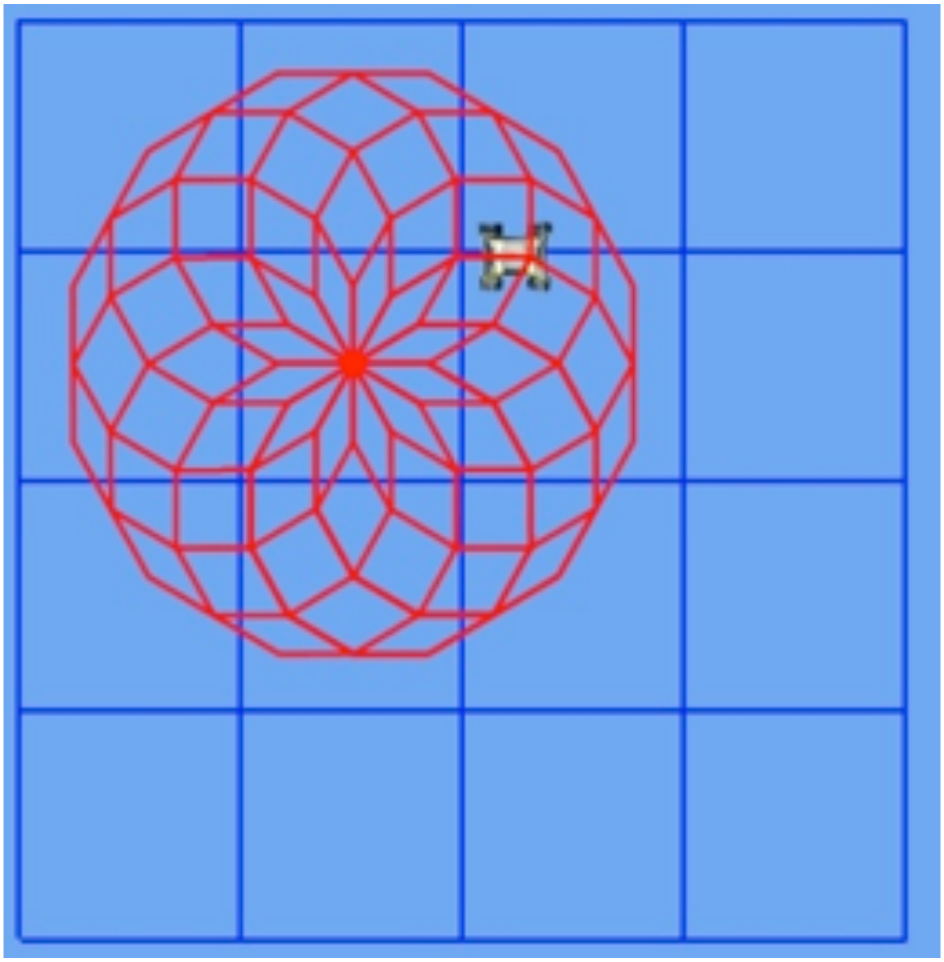

Figure 4: A computer graphics drawing of a multi-dodecagon.

The group then set the more difficult task of having a robot draw the design. Their result is shown in:

http://www.youtube.com/watch?feature=player_embedded&v=PwNZRaoQzmg

Applying Knowledge of Trigonometry
It is one thing to use the principles of trigonometry in a class when there are hints that the principles are needed. In this example (see Figure 5), one student recognized that something was needed, and then realized he were already familiar with it.

What makes this an interesting case, from the perspective of mathematics education, is that the student took time to describe the mathematical knowledge that he already knew, and then showed how he applied it to the particular real-world problems. [14]

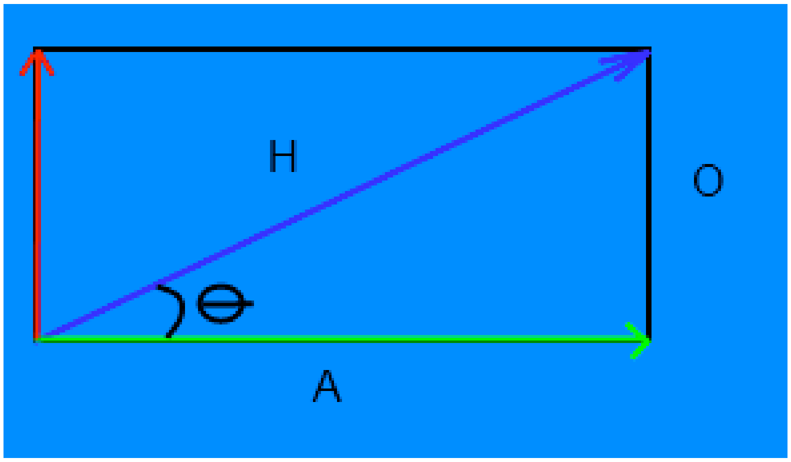

Figure 5: From a student's blog: a review of trigonometric properties.

http://cneuburg.wordpress.com/

Discussion

There are benefits to using robots, especially increased interest in the process. Robots work in the real world, and do not yield mathematically exact results. The mathematical problem provides a guide to the real world problem. Students learn that certain errors are the result of conceptual misconceptions. On other occasions, the real world measurement differs from the mathematical solution, then students can recognize that some results are close enough, and that errors always occur in working with actual materials. This requires much more involvement in the problem than students typically have. Students must understand both the mathematical problem and the real world problem. The robots require the student to monitor the success of the problem solving process, as the robot illustrates the solution to the problem. Also, students can see the accuracy of their solution by comparing the obtained solution with the actual location of the robots, which is the basis of the error checking process. Both monitoring and error checking enhance mathematical problem solving.

One of the engineering students saw mathematics in a distinctly new way. He realized that it was one thing to demonstrate the principles of trigonometry in a mathematics class, especially when there are hints that suggest the relevant principle. It is quite another thing to apply those principles in a novel real-world context. This student succeeded in a situation where difficulties are well-documented. This study provides examples of the smaller group of mathematical problem solving: successfully applying knowledge that was not immediately obvious.

The ultimate goal for each of these high school student teams is to draw and reproduce one simple geometric shape of their own choosing. The teams who achieve higher level of sophistication, reproducibility, and battery life will be judged to be superior. As can be imagined any number of criteria can be set – and it is to be expected that as the repertoire of available lessons and code bases increases, more intricate patterns and challenges will emerge.

This paper documents on-going work on building low cost robots that every K-12 school can afford. It is based on open source principles, making it easy to learn from the community, and to innovate and contribute back to the community. New ideas and lessons will evolve that can provide remuneration to the inventors, while as a larger community we will all make progress in educating our next generation in math and engineering principles.

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